5-7 Roots and Zeros

Solve each equation. State the number and type of roots.

1. $x^2 - 3x - 10 = 0$

SOLUTION:

$$x^2 - 3x - 10 = 0 \quad \text{Original equation}$$
$$(x-5)(x+2) = 0 \quad \text{Solve for } x$$
$$x = 5 \text{ or } x = -2$$

The equation has two real roots –2 and 5.

2. $x^3 + 12x^2 + 32x = 0$

SOLUTION:

$$x^3 + 12x^2 + 32x = 0 \quad \text{Original equation.}$$
$$x\left(x^2 + 12x + 32\right) = 0 \quad \text{Factor the GCF.}$$
$$x(x+8)(x+4) = 0 \quad \text{Factor binomial}$$
$$x = 0 \text{ or } x = -8 \text{ or } x = -4 \quad \text{Solve for } x$$

The equation has three real roots –8, –4 and 0.

3. $16x^4 - 81 = 0$

SOLUTION:

$$16x^4 - 81 = 0$$
$$\left(4x^2\right)^2 - 9^2 = 0$$
$$\left(4x^2 + 9\right)\left(4x^2 - 9\right) = 0$$

$$4x^2 + 9 = 0 \text{ or } 4x^2 - 9 = 0$$
$$x = \pm\sqrt{-\frac{9}{4}} \text{ or } x = \pm\sqrt{\frac{9}{4}}$$
$$x = -\frac{3}{2}i, \frac{3}{2}i \text{ or } x = -\frac{3}{2}, \frac{3}{2}$$

The equation has two real roots $-\frac{3}{2}, \frac{3}{2}$ and two imaginary roots $-\frac{3}{2}i, \frac{3}{2}i$.

4. $0 = x^3 - 8$

SOLUTION:

$x^3 - 8 = 0$	Original equation
$x^3 - 2^3 = 0$	Regroup to perfect cubes.
$(x-2)\left(x^2 + 2x + 4\right) = 0$	Factor the difference
$x = 2 \quad \text{or} \quad x^2 + 2x + 4 = 0$	Solve for x
$x = \dfrac{-2 \pm \sqrt{4-16}}{2}$	Use quadratic formula
$x = \dfrac{-2 \pm \sqrt{-12}}{2}$	Simplify under radical
$x = \dfrac{-2 \pm i2\sqrt{3}}{2}$	Simplify
$x = -1 \pm i\sqrt{3}$	Solve for x

The equation has one real root 2 and two imaginary roots $-1 + i\sqrt{3}$ and $-1 - i\sqrt{3}$.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

5. $f(x) = x^3 - 2x^2 + 2x - 6$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^3 - 2x^2 + 2x - 6$$

There are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros.

$$f(-x) = -x^3 - 2x^2 - 2x - 6$$

There are 0 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 0 negative real zeros.
Thus, $f(x)$ has 3 or 1 positive real zeros, 0 negative real zeros, 0 or 2 imaginary zeros.

6. $f(x) = 6x^4 + 4x^3 - x^2 - 5x - 7$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = 6x^4 + 4x^3 - x^2 - 5x - 7$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = 6x^4 - 4x^3 - x^2 + 5x - 7$$

There are 3 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 3 or 1 negative real zeros.
Thus, $f(x)$ has 1 positive real zeros, 1 or 3 negative real zeros, and 0 or 2 imaginary zeros.

7. $f(x) = 3x^5 - 8x^3 + 2x - 4$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = 3x^5 - 8x^3 + 2x - 4$$

There are 3 sign changes for the coefficients of $f(x)$, so the function has 3 or 1 positive real zeros.

$$f(-x) = -3x^5 + 8x^3 - 2x - 4$$

There are 2 sign changes for the coefficients of $f(-x)$, $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 1 or 3 positive real zeros, 0 or 2 negative real zeros, 0 or 2 or 4 imaginary zeros.

8. $f(x) = -2x^4 - 3x^3 - 2x - 5$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = -2x^4 - 3x^3 - 2x - 5$$

There are 0 sign changes for the coefficients of $f(x)$, so the function has 0 positive real zeros.

$$f(-x) = -2x^4 + 3x^3 + 2x - 5$$

There are 2 sign changes for the coefficients of $f(-x)$, $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 0 positive real zeros, 0 or 2 negative real zeros, 2 or 4 imaginary zeros.

5-7 Roots and Zeros

Find all zeros of each function.

9. $f(x) = x^3 + 9x^2 + 6x - 16$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^3 + 9x^2 + 6x - 16$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = -x^3 + 9x^2 - 6x - 16$$

There are 2 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrr} -2 & 1 & 9 & 6 & -16 \\ & 0 & -2 & -14 & 16 \\ \hline & 1 & 7 & -8 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 9x^2 + 6x - 16 \\ &= (x+2)(x^2 + 7x - 8) \\ &= (x+2)(x-1)(x+8) \end{aligned}$$

The function has zeros at –8, –2 and 1.

5-7 Roots and Zeros

10. $f(x) = x^3 + 7x^2 + 4x + 28$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^3 + 7x^2 + 4x + 28$$

There are 0 sign changes for the coefficients of $f(x)$, so the function has 0 positive real zeros.

$$f(-x) = -x^3 + 7x^2 - 4x + 28$$

There are 3 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 3 or 1 negative real zeros.
Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrr} -7 & 1 & 7 & 4 & 28 \\ & 0 & -7 & 0 & -28 \\ \hline & 1 & 0 & 4 & \vert 0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 7x^2 + 4x + 28 \\ &= (x + 7)(x^2 + 4) \end{aligned}$$

Since the depressed polynomial $x^2 + 4$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} \\ &= \frac{\pm\sqrt{-16}}{2} \\ &= 2i \text{ or } -2i \end{aligned}$$

The function has zeros at –7, –2i and 2i.

11. $f(x) = x^4 - 2x^3 - 8x^2 - 32x - 384$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^4 - 2x^3 - 8x^2 - 32x - 384$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = x^4 + 2x^3 - 8x^2 + 32x - 384$$

There are 3 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 3 or 1 negative real zeros.
Thus, $f(x)$ has 4 real zeros or 2 real zeros and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} -4 & 1 & -2 & -8 & -32 & -384 \\ & 0 & -4 & 24 & -64 & 384 \\ \hline & 1 & -6 & 16 & -96 & 0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 6x^2 + 16x - 96$ to find the second zero.

$$\begin{array}{r|rrrr} 6 & 1 & -6 & 16 & -96 \\ & 0 & 6 & 0 & 96 \\ \hline & 1 & 0 & 16 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x^4 - 2x^3 - 8x^2 - 32x - 384 \\ &= (x+4)(x-6)(x^2+16) \end{aligned}$$

The depressed polynomial $x^2 + 16$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(16)}}{2(1)} \\ &= \frac{\pm\sqrt{-64}}{2} \\ &= 4i \text{ or } -4i \end{aligned}$$

The function has zeros at -4, 6, $-4i$ and $4i$.

5-7 Roots and Zeros

12. $f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$$

There are 3 sign changes for the coefficients of $f(x)$, so the function has 3 or 1 positive real zero.

$$f(-x) = x^4 + 6x^3 + 9x^2 - 6x - 10$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 4 real zeros or 2 real zeros and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 9 & 6 & -10 \\ & 0 & 1 & -5 & 4 & 10 \\ \hline & 1 & -5 & 4 & 10 & |0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 5x^2 + 4x + 10$ to find a second zero.

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 4 & 10 \\ & 0 & -1 & 6 & -10 \\ \hline & 1 & -6 & 10 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^4 - 6x^3 + 9x^2 + 6x - 10 \\ &= (x-1)(x+1)(x^2 - 6x + 10) \end{aligned}$$

Since the depressed polynomial $x^2 - 6x + 10$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{6 \pm \sqrt{-4}}{2} \\ &= 3 + i \text{ or } 3 - i \end{aligned}$$

The function has zeros at $-1, 1, 3 - i$ and $3 + i$.

Write a polynomial function of least degree with integral coefficients that have the given zeros.

13. 4, –1, 6

SOLUTION:

Since 4, –1 and 6 are the zeros of the polynomial,
$x - 4$, $x + 1$ and $x - 6$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x-4)(x+1)(x-6) \\ &= (x^2-3x-4)(x-6) \\ &= x^3-3x^2-4x-6x^2+18x+24 \\ &= x^3-9x^2+14x+24 \end{aligned}$$

Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x)=x^3-9x^2+14x+24$ is a polynomial function of least degree with integral coefficients and zeros of 4, –1 and 6.

14. 3, –1, 1, 2

SOLUTION:

Since 3, –1, 1 and 2 are the zeros of the polynomial, $x - 3$, $x + 1$, $x - 1$ and $x - 2$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x-3)(x+1)(x-1)(x-2) \\ &= (x-3)(x-2)(x+1)(x-1) \\ &= (x^2-5x+6)(x^2-1) \\ &= x^4-5x^3+5x^2+5x-6 \end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x)=x^4-5x^3+5x^2+5x-6$ is a polynomial function of least degree with integral coefficients and zeros of 3, –1, 1 and 2.

15. –2, 5, –3***i***

SOLUTION:

If $-3i$ is a zero, then $3i$ is also a zero according to the Complex Conjugates Theorem.
So $x + 2$, $x - 5$, $x - 3i$ and $x + 3i$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x+2)(x-5)(x-3i)(x+3i) \\ &= (x^2-3x-10)(x^2+9) \\ &= x^4-3x^3-x^2-27x-90 \end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x)=x^4-3x^3-x^2-27x-90$ is a polynomial function of least degree with integral coefficients and zeros of –2, 5, –3*i*, and 3*i*.

16. –4, 4 + ***i***

SOLUTION:
If 4 + *i* is a zero, then 4 – *i* is also a zero according to the Complex Conjugates Theorem.
So $x + 4$, $x - (4 + i)$ and $x - (4 - i)$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x+4)[x-(4+i)][x-(4-i)] \\ &= (x+4)[(x-4)-i][(x-4)+i] \\ &= (x+4)\left[(x-4)^2 - i^2\right] \\ &= (x+4)\left[x^2 - 8x + 17\right] \\ &= x^3 - 4x^2 - 15x + 68 \end{aligned}$$

Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x) = x^3 - 4x^2 - 15x + 68$ is a polynomial function of least degree with integral coefficients and zeros of –4, 4 + *i* and 4 – *i*.

Solve each equation. State the number and type of roots.

17. $2x^2 + x - 6 = 0$

SOLUTION:

$$\begin{aligned} 2x^2 + x - 6 &= 0 \\ (2x-3)(x+2) &= 0 \\ x = \frac{3}{2} \quad \text{or} \quad x &= -2 \end{aligned}$$

The equation has two real roots, –2 and $\frac{3}{2}$.

18. $4x^2 + 1 = 0$

SOLUTION:

$$\begin{aligned} 4x^2 + 1 &= 0 \\ x^2 &= -\frac{1}{4} \\ x &= \pm\sqrt{-\frac{1}{4}} \\ x = -\frac{1}{2}i \quad &\text{or} \quad x = \frac{1}{2}i \end{aligned}$$

The equation has two imaginary roots, $-\frac{1}{2}i$ and $\frac{1}{2}i$.

19. $x^3 + 1 = 0$

SOLUTION:

$$
\begin{aligned}
x^3 + 1 &= 0 \\
(x+1)(x^2 - x + 1) &= 0 \\
x + 1 = 0 \quad \text{or} \quad x^2 - x + 1 &= 0 \\
x = -1 \quad \text{or} \quad x &= \frac{1 \pm i\sqrt{3}}{2}
\end{aligned}
$$

The equation has 1 real root, –1, and 2 imaginary roots, $\frac{1 \pm i\sqrt{3}}{2}$.

20. $2x^2 - 5x + 14 = 0$

SOLUTION:

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(14)}}{2(2)} \\
&= \frac{5 \pm i\sqrt{87}}{4}
\end{aligned}
$$

The equation has two imaginary roots, $\frac{5 \pm i\sqrt{87}}{4}$.

21. $-3x^2 - 5x + 8 = 0$

SOLUTION:

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-3)(8)}}{2(-3)} \\
&= \frac{5 \pm 11}{-6} \\
&= -\frac{8}{3} \quad \text{or} \quad 1
\end{aligned}
$$

The equation has two real roots, $-\frac{8}{3}$ and 1.

22. $8x^3 - 27 = 0$

SOLUTION:

$$(2x)^3 - 3^3 = 0$$
$$(2x-3)(4x^2+6x+9) = 0$$
$$2x-3=0 \quad \text{or} \quad 4x^2+6x+9=0$$
$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{-3 \pm 3i\sqrt{3}}{4}$$

The equation has 1 real root, $\frac{3}{2}$, and 2 imaginary roots, $\frac{-3 \pm 3i\sqrt{3}}{4}$.

23. $16x^4 - 625 = 0$

SOLUTION:

$$16x^4 - 625 = 0$$
$$\left((2x)^2\right)^2 - \left(5^2\right)^2 = 0$$
$$\left((2x)^2 - 5^2\right)\left((2x)^2 + 5^2\right) = 0$$
$$(2x+5)(2x-5)(4x^2+25) = 0$$
$$2x+5=0 \quad \text{or} \quad 2x-5=0 \quad \text{or} \quad 4x^2+25=0$$
$$x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2} \quad \text{or} \quad x = \pm\frac{5}{2}i$$

The equation has two real roots and two imaginary roots.

24. $x^3 - 6x^2 + 7x = 0$

SOLUTION:

$$x^3 - 6x^2 + 7x = 0$$
$$x(x^2 - 6x + 7) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 6x + 7 = 0$$
$$x = 3 \pm \sqrt{2}$$

The equation has three real roots: 0, $3+\sqrt{2}$ and $3-\sqrt{2}$.

25. $x^5 - 8x^3 + 16x = 0$

SOLUTION:

$$x^5 - 8x^3 + 16x = 0$$
$$x(x^4 - 8x^2 + 16) = 0$$
$$x = 0 \quad \text{or} \quad x = \pm 2, \pm 2$$

The equation has 5 real roots: –2, –2, 0, 2 and 2.

5-7 Roots and Zeros

26. $x^5 + 2x^3 + x = 0$

SOLUTION:

$$x^5 + 2x^3 + x = 0$$
$$x\left(x^4 + 2x^2 + 1\right) = 0$$
$$x = 0 \quad \text{or} \quad x = \pm i, \pm i$$

The equation has 1 real root, 0, and 4 imaginary roots: $-i$, $-i$, i, and i.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

27. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = x^4 + 5x^3 + 2x^2 - 5x + 7$$

There are 2 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 0 or 2 positive real zeros, 0 or 2 negative real zeros, 0 or 2 or 4 imaginary zeros.

28. $f(x) = 2x^3 - 7x^2 - 2x + 12$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = 2x^3 - 7x^2 - 2x + 12$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = -2x^3 - 7x^2 + 2x + 12$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 2 or 0 positive real zeros, 1 negative real zero, 0 or 2 imaginary zeros.

29. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = -3x^5 + 5x^4 + 4x^2 - 8$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = 3x^5 + 5x^4 + 4x^2 - 8$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 2 or 0 positive real zeros, 1 negative real zero, 2 or 4 imaginary zeros.

30. $f(x) = x^4 - 2x^2 - 5x + 19$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^4 - 2x^2 - 5x + 19$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = x^4 - 2x^2 + 5x + 19$$

There are 2 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 0 or 2 positive real zeros, 0 or 2 negative real zeros, 0 or 2 or 4 imaginary zeros.

31. $f(x) = 4x^6 - 5x^4 - x^2 + 24$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = 4x^6 - 5x^4 - x^2 + 24$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = 4x^6 - 5x^4 - x^2 + 24$$

There are 2 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 0 or 2 positive real zeros, 0 or 2 negative real zeros, 2 or 4 or 6 imaginary zeros.

32. $f(x) = -x^5 + 14x^3 + 18x - 36$

SOLUTION:

Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = -x^5 + 14x^3 + 18x - 36$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zeros.

$$f(-x) = x^5 - 14x^3 - 18x - 36$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 0 or 2 positive real zeros, 1 negative real zeros, 2 or 4 imaginary zeros.

Find all zeros of each function.

33. $f(x) = x^3 + 7x^2 + 4x - 12$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^3 + 7x^2 + 4x - 12$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = -x^3 + 7x^2 - 4x - 12$$

There are 2 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 2 or 0 negative real zeros.
Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrr} 1 & 1 & 7 & 4 & -12 \\ & 0 & 1 & 8 & 12 \\ \hline & 1 & 8 & 12 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + 7x^2 + 4x - 12 \\ &= (x-1)(x^2 + 8x + 12) \\ &= (x-1)(x+6)(x+2) \end{aligned}$$

The function has zeros at –6, –2 and 1.

34. $f(x) = x^3 + x^2 - 17x + 15$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^3 + x^2 - 17x + 15$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 2 or 0 positive real zero.

$$f(-x) = -x^3 + x^2 + 17x + 15$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -17 & 15 \\ & 0 & 1 & 2 & -15 \\ \hline & 1 & 2 & -15 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^3 + x^2 - 17x + 15 \\ &= (x-1)(x^2 + 2x - 15) \\ &= (x-1)(x+5)(x-3) \end{aligned}$$

The function has zeros at –5, 1 and 3.

35. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = x^4 + 3x^3 - 3x^2 + 75x - 700$$

There are 3 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 3 or 1 negative real zeros.
Thus, $f(x)$ has 4 real zeros or 2 real zeros and 2 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} -4 & 1 & -3 & -3 & -75 & -700 \\ & 0 & -4 & 28 & -100 & 700 \\ \hline & 1 & -7 & 25 & -175 & |0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 7x^2 + 25x - 175$ to find a second zero.

$$\begin{array}{r|rrrr} 7 & 1 & -7 & 25 & -175 \\ & 0 & 7 & 0 & 175 \\ \hline & 1 & 0 & 25 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^4 - 3x^3 - 3x^2 - 75x - 700 \\ &= (x+4)(x-7)(x^2+25) \end{aligned}$$

The depressed polynomial $x^2 + 25$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)} \\ &= \frac{\pm\sqrt{-100}}{2} \\ &= 5i \text{ or } -5i \end{aligned}$$

The function has zeros at -4, 7, $-5i$ and $5i$.

36. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$$

There is no sign change for the coefficients of $f(x)$, so the function has 0 positive real zero.

$$f(-x) = x^4 - 6x^3 + 73x^2 - 384x + 576$$

There are 4 sign changes for the coefficients of $f(-x)$, so $f(x)$ has 4, 2 or 0 negative real zeros.
Thus, $f(x)$ has 4 real zeros or 2 real zeros and 2 imaginary zeros or 4 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 73 & 384 & 576 \\ & 0 & -3 & -9 & -192 & -576 \\ \hline & 1 & 3 & 64 & 192 & |0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 + 3x^2 + 64x + 192$ to find a second zero.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 64 & 192 \\ & 0 & -3 & 0 & -192 \\ \hline & 1 & 0 & 64 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^4 + 6x^3 + 73x^2 + 384x + 576 \\ &= (x+3)(x+3)(x^2+64) \end{aligned}$$

The depressed polynomial $x^2 + 64$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(64)}}{2(1)} \\ &= \frac{\pm\sqrt{-256}}{2} \\ &= 8i \text{ or } -8i \end{aligned}$$

The function has zeros at –3, –3, –8i and 8i.

37. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$$

There are 4 sign changes for the coefficients of $f(x)$, so the function has 4, 2 or 0 positive real zero.

$$f(-x) = x^4 + 8x^3 + 20x^2 + 32x + 64$$

There is 0 sign change for the coefficients of $f(-x)$, so $f(x)$ has 0 negative real zeros.
Thus, $f(x)$ has 4 real zeros, or 2 real zeros and 2 imaginary zeros, or 4 imaginary zeros.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} 4 & 1 & -8 & 20 & -32 & 64 \\ & 0 & 4 & -16 & 16 & -64 \\ \hline & 1 & -4 & 4 & -16 & |0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 4x^2 + 4x - 16$ to find a second zero.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & 0 & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & |0 \end{array}$$

$$\begin{aligned} f(x) &= x^4 - 8x^3 + 20x^2 - 32x + 64 \\ &= (x-4)(x-4)(x^2+4) \end{aligned}$$

The depressed polynomial $x^2 + 4$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} \\ &= \frac{\pm\sqrt{-16}}{2} \\ &= 2i \text{ or } -2i \end{aligned}$$

The function has zeros at 4, 4, $-2i$ and $2i$.

38. $f(x) = x^5 - 8x^3 - 9x$

SOLUTION:

First, determine the total number of zeros.

$$f(x) = x^5 + 0x^4 - 8x^3 + 0x^2 - 9x + 0$$

There is 1 sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$f(-x) = -x^5 + 8x^3 + 9x$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 2 real zeros, 0 and 2 imaginary roots.

$$\begin{aligned} f(x) &= x^5 - 8x^3 - 9x \\ &= x(x^4 - 8x^2 - 9) \end{aligned}$$

Therefore, $x = 0$ is one of the zeros of the function $f(x)$.
Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -8 & 0 & -9 \\ & 0 & -3 & 9 & -3 & 9 \\ \hline & 1 & -3 & 1 & -3 & |0 \end{array}$$

Use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 3x^2 + x - 3$ to find a second zero.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & 0 & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & |0 \end{array}$$

The depressed polynomial $x^2 + 1$ is quadratic, use the Quadratic Formula to find the remaining zeros.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} \\ &= \frac{\pm\sqrt{-4}}{2} \\ &= i \text{ or } -i \end{aligned}$$

The function has zeros at -3, 0, 3, $-i$ and i.

Write a polynomial function of least degree with integral coefficients that have the given zeros.

39. 5, –2, –1

SOLUTION:

$x - 5$, $x + 2$ and $x + 1$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x-5)(x+2)(x+1) \\ &= (x^2-3x-10)(x+1) \\ &= x^3-2x^2-13x-10 \end{aligned}$$

Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x) = x^3 - 2x^2 - 13x - 10$ is a polynomial function of least degree with integral coefficients and zeros of 5, –2 and –1.

40. –4, –3, 5

SOLUTION:

$x + 4$, $x + 3$ and $x - 5$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x+4)(x+3)(x-5) \\ &= (x^2+7x+12)(x-5) \\ &= x^3+2x^2-23x-60 \end{aligned}$$

Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x) = x^3 + 2x^2 - 23x - 60$ is a polynomial function of least degree with integral coefficients and zeros of –4, –3 and 5.

41. –1, –1, 2***i***

SOLUTION:

If 2*i* is a zero, then –2*i* is also a zero according to the Complex Conjugates Theorem.
So $x + 1$, $x + 1$, $x - 2i$ and $x + 2i$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x+1)(x+1)(x-2i)(x+2i) \\ &= (x^2+2x+1)\left(x^2-(2i)^2\right) \\ &= (x^2+2x+1)(x^2+4) \\ &= x^4+2x^3+5x^2+8x+4 \end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x) = x^4 + 2x^3 + 5x^2 + 8x + 4$ is a polynomial function of least degree with integral coefficients and zeros of –1, –1, –2*i*, and 2*i*.

5-7 Roots and Zeros

42. –3, 1, –3***i***

SOLUTION:

If $-3i$ is a zero, then $3i$ is also a zero according to the Complex Conjugates Theorem.
So $x + 3$, $x - 1$, $x - 3i$ and $x + 3i$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned}P(x) &= (x+3)(x-1)(x-3i)(x+3i) \\ &= \left(x^2+2x-3\right)\left(x^2+9\right) \\ &= x^4+2x^3+6x^2+18x-27\end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x) = x^4 + 2x^3 + 6x^2 + 18x - 27$ is a polynomial function of least degree with integral coefficients and zeros of –3, 1, $-3i$, and $3i$.

43. 0, –5, 3 + ***i***

SOLUTION:

If $3 + i$ is a zero, then $3 - i$ is also a zero according to the Complex Conjugates Theorem.
So $x - 0$, $x + 5$, $x - (3 + i)$ and $x + (3 - i)$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned}P(x) &= (x-0)(x+5)\left[x-(3+i)\right]\left[x-(3-i)\right] \\ &= \left(x^2+5x\right)\left[(x-3)-i\right]\left[(x-3)+i\right] \\ &= \left(x^2+5x\right)\left[(x-3)^2-i^2\right] \\ &= \left(x^2+5x\right)\left[x^2-6x+10\right] \\ &= x^4-x^3-20x^2+50x\end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x) = x^4 - x^3 - 20x^2 + 50x$ is a polynomial function of least degree with integral coefficients and zeros of 0, –5, $3 + i$ and $3 - i$.

44. $-2, -3, 4 - 3i$

SOLUTION:

If $4 - 3i$ is a zero, then $4 + 3i$ is also a zero according to the Complex Conjugates Theorem.
So $x + 2$, $x + 3$, $x - (4 - 3i)$ and $x - (4 + 3i)$ are factors of the polynomial.
Write the polynomial function as a product of its factors.

$$\begin{aligned} P(x) &= (x+2)(x+3)[x-(4-3i)][x-(4+3i)] \\ &= (x^2+5x+6)[(x-4)+3i][(x-4)-3i] \\ &= (x^2+5x+6)\left[(x-4)^2-(3i)^2\right] \\ &= (x^2+5x+6)\left[x^2-8x+25\right] \\ &= x^4-3x^3-9x^2+77x+150 \end{aligned}$$

Because there are 4 zeros, the degree of the polynomial function must be 4, so $P(x) = x^4 - 3x^3 - 9x^2 + 77x + 150$ is a polynomial function of least degree with integral coefficients and zeros of -2, -3, $4 - 3i$ and $4 + 3i$.

45. **CCSS REASONING** A computer manufacturer determines that for each employee, the profit for producing x computers per day is $P(x) = -0.006x^4 + 0.15x^3 - 0.05x^2 - 1.8x$.
a. How many positive real zeros, negative real zeros, and imaginary zeros exist?
b. What is the meaning of the zeros in this situation?

SOLUTION:

a.
Examine the number of sign changes for $f(x)$ and $f(-x)$.
$P(x) = -0.006x^4 + 0.15x^3 - 0.05x^2 - 1.8x$
There are 2 sign changes for the coefficients of $f(x)$, the function has 2 or 0 positive real zeros.
$P(-x) = -0.006x^4 - 0.15x^3 - 0.05x^2 + 1.8x$
There is 1 sign change for the coefficients of $f(-x)$, $f(x)$ has 1 negative real zeros.
Thus, $f(x)$ has 3 real zeros and 1 imaginary zero or 1 real zero and 3 imaginary zeros.
b.
Nonnegative roots represent numbers of computers produced per day which lead to no profit for the manufacturer.

Sketch the graph of each function using its zeros.

46. $f(x) = x^3 - 5x^2 - 2x + 24$

SOLUTION:

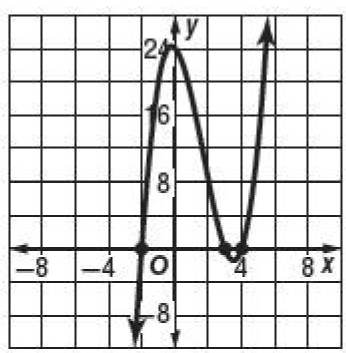
Because $f(x)$ is of degree 3, there are at most 3 zeros. For $f(x)$, there is one sign change, so there are 1 or 0 positive real zeros. For $f(-x)$, there are two sign changes, so there are 2 or 0 negative real zeros. You can find the zeros by factoring, dividing, or using synthetic division.
List some possible negative values, and then use synthetic substitution to evaluate $f(x)$ for real values of x.

x	1	-5	-2	24
-3	1	-8	22	42
-2	1	-7	12	0
-1	1	-6	4	20

From the table, one negative real zero is $x = -2$. The depressed polynomial $x^2 - 7x + 12$ can be factored as $(x - 3)(x - 4)$. So, there are two positive real zeros $x = 3$ and $x = 4$.

The zeros are -2, 3, and 4. Plot all three zeros on a coordinate plane. You need to determine what happens to the graph between the zeros and at its extremes. Use a table to analyze values close to the zeros. Use these values to complete the graph.

x	-3	0	$\frac{7}{2}$	5
y	-42	0.4375	-2	112



47. $f(x)=4x^3+2x^2-4x-2$

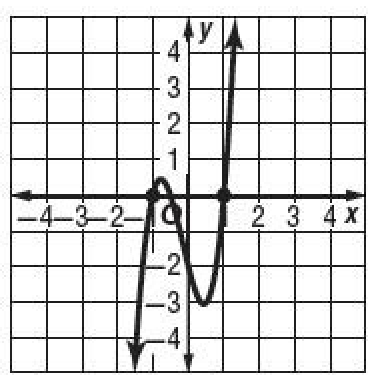
SOLUTION:

Because $f(x)$ is of degree 3, there are at most 3 zeros. For $f(x)$, there is one sign change, so there are 1 or 0 positive real zeros. For $f(-x)$, there are two sign changes, so there are 2 or 0 negative real zeros. You can find the zeros by factoring, dividing, or using synthetic division.

$$\begin{aligned}f(x)&=4x^3+2x^2-4x-2\\&=2x^2(2x+1)-2(2x+1)\\&=(2x^2-2)(2x+1)\\&=2(x^2-1)(2x+1)\end{aligned}$$

The zeros are -1, $-\frac{1}{2}$, and 1. Plot all three zeros on a coordinate plane. You need to determine what happens to the graph between the zeros and at its extremes. Use a table to analyze values close to the zeros. Use these values to complete the graph.

x	-2	$-\frac{3}{4}$	0	3
y	-18	0.4375	-2	112



48. $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

SOLUTION:

Because $f(x)$ is of degree 4, there are at most 4 zeros. For $f(x)$, there are 3 sign changes, so there are 3, 1, or 0 positive real roots. For $f(-x)$, there is 1 sign change, so there is 1 or 0 negative real zero. You can find the zeros by factoring, dividing, or synthetic division.
List some possible negative values, and then use synthetic substitution to evaluate $f(x)$ for real values of x.

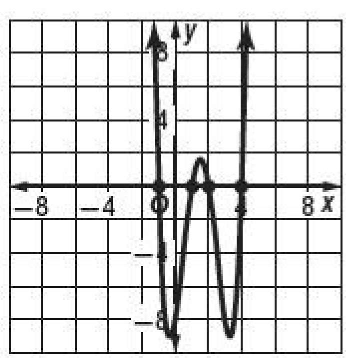
x	1	-6	7	6	-8
-4	1	-10	-33	138	-560
-3	1	-9	34	-96	280
-2	1	-8	23	-40	72
-1	1	-7	14	-8	0

From the table, one negative real zero is $x = -1$. The depressed polynomial is $x^3 - 7x^2 + 14x - 8$. List some possible positive values, and then use synthetic substitution to evaluate the depressed polynomial for real values of x.

x	1	-7	14	-8
1	1	-6	8	0
2	1	-5	4	0
3	1	-4	2	-2
4	1	-3	2	0

There are three positive real zeros $x = 1$, $x = 2$, and $x = 4$. Plot all of the zeros on a coordinate plane. You need to determine what happens to the graph between the zeros and at its extremes. Use a table to analyze values close to the zeros. Use these zeros to complete the graph.

x	-2	0	$\frac{3}{2}$	3	5
y	72	-8	1.5625	-8	72



49. $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

SOLUTION:

Because $f(x)$ is of degree 4, there are at most 4 zeros. For $f(x)$, there are 3 sign changes, so there are 3, 1, or 0 positive real roots. For $f(-x)$, there is 1 sign change, so there is 1 or 0 negative real zero. You can find the zeros by factoring, dividing, or synthetic division.
List some possible negative values, and then use synthetic substitution to evaluate $f(x)$ for real values of x.

x	1	-6	9	4	-12
–4	1	–10	49	–192	756
–3	1	–9	36	–104	300
–2	1	–8	25	–46	80
–1	1	–7	16	–12	0

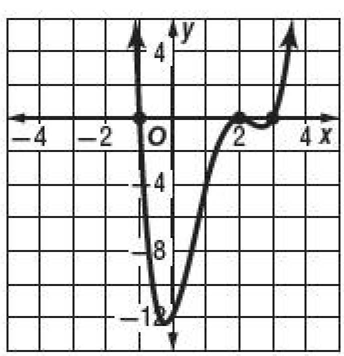
From the table, one negative real zero is $x = -1$. The depressed polynomial is $x^3 - 7x^2 + 16x - 12$. List some possible positive values, and then use synthetic substitution to evaluate the depressed polynomial for real values of x.

x	1	-7	16	-12
1	1	–6	10	-2
2	1	–5	6	0
3	1	–4	4	0
4	1	–3	4	4

There are two positive real zeros $x = 2$ and $x = 3$. The depressed polynomial for $x = 3$, $x^2 - 4x + 4$, can be factored as $(x - 2)^2$. So, $x = 2$ is a double zero.

Plot all of the zeros on a coordinate plane. You need to determine what happens to the graph between the zeros and at its extremes. Use a table to analyze values close to the zeros. Use these zeros to complete the graph.

x	-2	0	$\frac{5}{2}$	4
y	80	-12	-0.4375	20

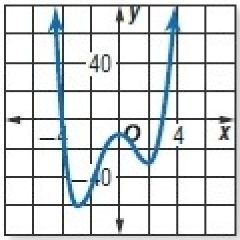


Match each graph to the given zeros.

a. –3, 4, *i*, –*i*

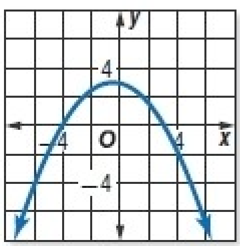
b. –4, 3

c. –4, 3, *i*, –*i*

50. 

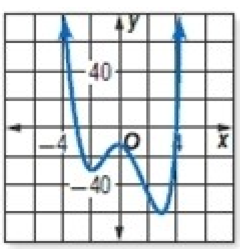
SOLUTION:

The graph intersects the *x*-axis at –4 and 3. Also, the graph has 3 turning points. So, the function representing the graph is of degree 4. The graph matches with the zeros in part c.

51. 

SOLUTION:

The graph intersects the *x*-axis at –4 and 3. Also, the graph has 1 turning point. So, the function representing the graph is of degree 2. The graph matches with the zeros in part b.

52. 

SOLUTION:

The graph intersects the *x*-axis at –3 and 4. Also, the graph has 3 turning points. So, the function representing the graph is of degree 4. The graph matches with the zeros in part a.

53. **CONCERTS** The amount of money Hoshi's Music Hall took in from 2003 to 2010 can be modeled by $M(x) = -2.03x^3 + 50.1x^2 - 214x + 4020$, where x is the years since 2003.
a. How many positive real zeros, negative real zeros, and imaginary zeros exist?
b. Graph the function using your calculator.
c. Approximate all real zeros to the nearest tenth. What is the significance of each zero in the context of the situation?

SOLUTION:

a.

Examine the number of sign changes for $M(x)$ and $M(-x)$.

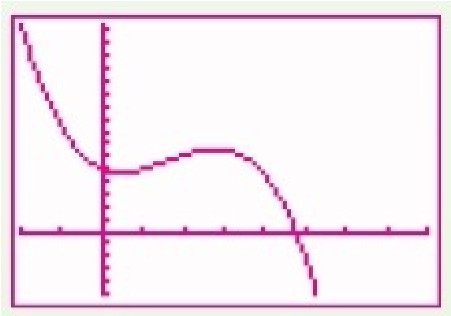
$$M(x) = -2.03x^3 + 50.1x^2 - 214x + 4020$$

There are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros.

$$M(-x) = 2.03x^3 + 50.1x^2 + 214x + 4020$$

There is 0 sign change for the coefficients of $f(-x)$, $f(x)$ has 0 negative real zeros.
Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.

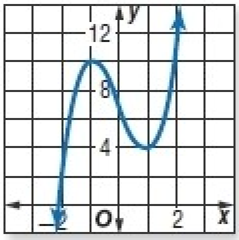
b.



[–10, 40] scl: 5 by [–4000, 13,200]
scl: 100

c. 23.8; Sample answer: According to the model, the music hall will not earn any money after 2026.

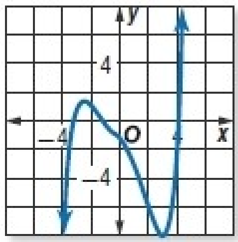
Determine the number of positive real zeros, negative real zeros, and imaginary zeros for each function. Explain your reasoning.

54. 

degree: 3

SOLUTION:

0 positive, 1 negative, 2 imaginary;
Sample answer: The graph does not cross the positive x-axis, and crosses the negative x-axis once.
Because the degree of the polynomial is 3, there are 3 – 1 or 2 imaginary zeros.

55. 

degree:5

SOLUTION:

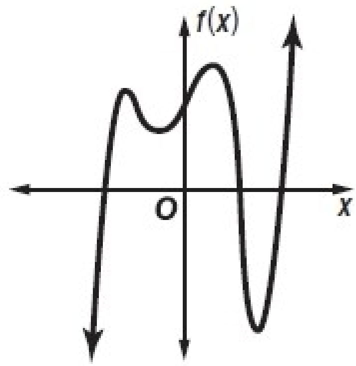
1 positive, 2 negative, 2 imaginary;
Sample answer: The graph crosses the positive *x*-axis once, and crosses the negative *x*-axis twice. Because the degree of the polynomial is 5, there are 5 – 3 or 2 imaginary zeros.

56. **OPEN ENDED** Sketch the graph of a polynomial function with:

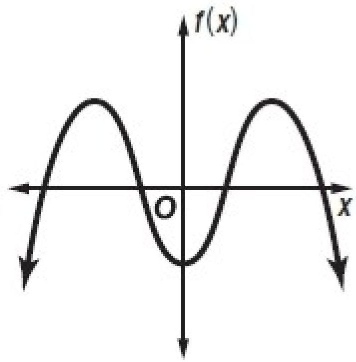
a. 3 real, 2 imaginary zeros
b. 4 real zeros
c. 2 imaginary zeros

SOLUTION:

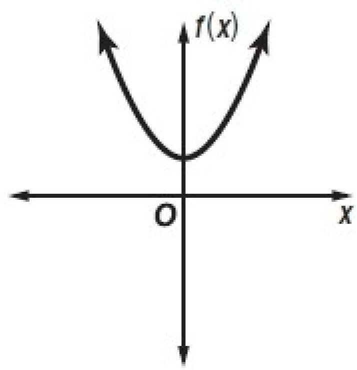
a. Sample answer: this polynomial function has a degree of 5 so the end behavior is in opposite directions. Since three of the zeros are real, the graph should intersect the *x*-axis three times.



b. Sample answer: this polynomial function has a degree of 4 so the end behavior is in the same direction. Since each of the four zeros are real, the graph should intersect the *x*-axis four times.



c. Sample answer: this polynomial function has a degree of 2 so the end behavior is in the same direction. Since each of the two zeros are imaginary, the graph should not intersect the *x*-axis.



57. **CHALLENGE** Write an equation in factored form of a polynomial function of degree 5 with 2 imaginary zeros, 1 non integral zero, and 2 irrational zeros. Explain.

SOLUTION:

Sample answer: $f(x) = (x+2i)(x-2i)(3x+5)\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)$ Use conjugates for the imaginary and irrational values.

58. **CCSS ARGUMENTS** Determine which equation is not like the others, Explain

$\boxed{r^4 + 1 = 0}$ $\boxed{r^3 + 1 = 0}$

$\boxed{r^2 - 1 = 0}$ $\boxed{r^3 - 8 = 0}$

SOLUTION:

$r^4 + 1 = 0$; Sample answer: The equation has imaginary solutions and all of the others have real solutions.

59. **REASONING** Provide a counter example for each statement.
a. All polynomial functions of degree greater than 2 have at least 1 negative real root.
b. All polynomial functions of degree greater than 2 have at least 1 positive real root.

SOLUTION:

Sample answer:
a. $f(x) = x^4 + 4x^2 + 4$
b. $f(x) = x^3 + 6x^2 + 9x$

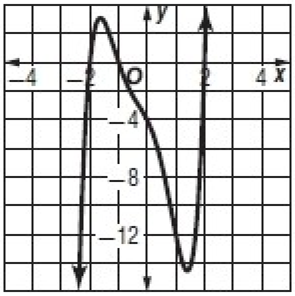
60. **WRITING IN MATH** Explain to a friend how you would use Descartes' Rule of Signs to determine the number of possible positive real roots and the number of possible negative roots of the polynomial function $f(x) = x^4 - 2x^3 + 6x^2 + 5x - 12$.

SOLUTION:

Sample answer: To determine the number of positive real roots, determine how many time the signs change in the polynomial as you move from left to right. In this function there are 3 changes in sign. Therefore, there may be 3 or 1 positive real roots. To determine the number of negative real roots, I would first evaluate the polynomial for $-x$. All of the terms with an odd-degree variable would change signs. Then I would again count the number of sign changes as I move from left to right. There would be only one change. Therefore there may be 1 negative root.

5-7 Roots and Zeros

61. Use the graph of the polynomial function below. Which is not a factor of the polynomial $x^5 + x^4 - 3x^3 - 3x^2 - 4x - 4$?



A $x - 2$
B $x + 2$
C $x - 1$
D $x + 1$

SOLUTION:

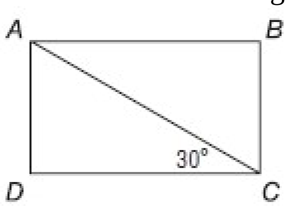
The graph intersects the *x*-axis at $x = -2, -1$ and 2. So, the factors of the function are $(x + 2)$, $(x + 1)$ and $(x - 2)$. $(x - 1)$ is not a factor of the polynomial.
C is the correct option.

62. **SHORT RESPONSE** A window is in the shape of an equilateral triangle. Each side of the triangle is 8 feet long. The window is divided in half by a support from one vertex to the midpoint of the side of the triangle opposite the vertex. Approximately how long is the support?

SOLUTION:

The support divides the triangles into two congruent triangles. The angles in each triangle are in the ratio $1:\sqrt{3}:2.$ Thus, the length of the support is $4\sqrt{3}$ feet, which is approximately 6.9 feet.

63. **GEOMETRY** In rectangle *ABCD*, $\overline{AD}$ is 8 units long. What is the length of $\overline{AB}$?



F 4 units
G 8 units
H $8\sqrt{3}$ units
J 16 units

SOLUTION:

Consider the triangle *CAD*. The sides are in the ratio $1:\sqrt{3}:2.$ Thus, the length of $\overline{DC}$ is $8\sqrt{3}$ units.

64. **SAT/ACT** The total area of a rectangle is $25a^4 - 16b^2$ square units. Which factors could represent the length and width?

A $(5a^2 + 4b)$ units and $(5a^2 + 4b)$ units
B $(5a^2 + 4b)$ units and $(5a^2 - 4b)$ units
C $(5a^2 - 4b)$ units and $(5a^2 - 4b)$ units
D $(5a - 4b)$ units and $(5a - 4b)$ units
E $(5a + 4b)$ units and $(5a - 4b)$ units

SOLUTION:

$$\begin{aligned} 25a^4 - 16b^2 &= (5a^2)^2 - (4b)^2 \\ &= (5a^2 - 4b)(5a^2 + 4b) \end{aligned}$$

Thus, B is the correct choice.

Use synthetic substitution to find $f(-8)$ and $f(4)$ for each function.

65. $f(x) = 4x^3 + 6x^2 - 3x + 2$

SOLUTION:

Divide the function by $x + 8$.

$$\begin{array}{r|rrrr} -8 & 4 & 6 & -3 & 2 \\ & 0 & -32 & 208 & -1640 \\ \hline & 4 & -26 & 205 & -1638 \end{array}$$

The remainder is –1638. Therefore, $f(-8) = -1638$.
Divide the function by $x - 4$.

$$\begin{array}{r|rrrr} 4 & 4 & 6 & -3 & 2 \\ & 0 & 16 & 88 & 340 \\ \hline & 4 & 22 & 85 & 342 \end{array}$$

The remainder is 342. Therefore, $f(4) = 342$.

66. $f(x) = 5x^4 - 2x^3 + 4x^2 - 6x$

SOLUTION:

Divide the function by $x + 8$.

$$\begin{array}{r|rrrrr} -8 & 5 & -2 & 4 & -6 & 0 \\ & 0 & -40 & 336 & -2720 & 21808 \\ \hline & 5 & -42 & 340 & -2726 & |21808 \end{array}$$

The remainder is 21808. Therefore, $f(-8) = 21808$.
Divide the function by $x - 4$.

$$\begin{array}{r|rrrrr} 4 & 5 & -2 & 4 & -6 & 0 \\ & 0 & 20 & 72 & 304 & 1192 \\ \hline & 5 & 18 & 76 & 298 & |1192 \end{array}$$

The remainder is 1192. Therefore, $f(4) = 1192$.

67. $f(x) = 2x^5 - 3x^3 + x^2 - 4$

SOLUTION:

Divide the function by $x + 8$.

$$\begin{array}{r|rrrrrr} -8 & 2 & 0 & -3 & 1 & 0 & -4 \\ & 0 & -16 & 128 & -1000 & 7992 & -63936 \\ \hline & 2 & -16 & 125 & -999 & 7992 & |-63940 \end{array}$$

The remainder is –63940. Therefore, $f(-8) = -63940$.
Divide the function by $x - 4$.

$$\begin{array}{r|rrrrrr} 4 & 2 & 0 & -3 & 1 & 0 & -4 \\ & 0 & 8 & 32 & 116 & 468 & 1872 \\ \hline & 2 & 8 & 29 & 117 & 468 & |1868 \end{array}$$

The remainder is 1868. Therefore, $f(4) = 1868$.

Factor completely. If the polynomial is not factorable, write prime.

68. $x^6 - y^6$

SOLUTION:

$$\begin{aligned} x^6 - y^6 &= \left(x^3\right)^2 - \left(y^3\right)^2 \\ &= \left(x^3 + y^3\right)\left(x^3 - y^3\right) \\ &= (x + y)\left(x^2 - xy + y^2\right)(x - y)\left(x^2 + xy + y^2\right) \end{aligned}$$

69. $a^6 + b^6$

SOLUTION:

$$a^6 + b^6 = \left(a^2\right)^3 + \left(b^2\right)^3$$
$$= \left(a^2 + b^2\right)\left(a^4 - a^2b^2 + b^4\right)$$

70. $4x^2y + 8xy + 16y - 3x^2z - 6xz - 12z$

SOLUTION:

$$4x^2y + 8xy + 16y - 3x^2z - 6xz - 12z$$
$$= 4y\left(x^2 + 2x + 4\right) - 3z\left(x^2 + 2x + 4\right)$$
$$= \left(x^2 + 2x + 4\right)(4y - 3z)$$

71. $5a^3 - 30a^2 + 40a + 2a^2b - 12ab + 16b$

SOLUTION:

$$5a^3 - 30a^2 + 40a + 2a^2b - 12ab + 16b$$
$$= 5a\left(a^2 - 6a + 8\right) + 2b\left(a^2 - 6a + 8\right)$$
$$= \left(a^2 - 6a + 8\right)(5a + 2b)$$
$$= (a - 4)(a - 2)(5a + 2b)$$

72. **BUSINESS** A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall.

a. $-8.1r^2 + 46.9r - 38.2 = 0$

b. $-8.1r^2 + 46.9r - 38.2 > 0$

c. $-8.1r^2 + 46.9r - 38.2 > 10$

d. $-8.1r^2 + 46.9r - 38.2 < 10$

SOLUTION:

a. Use the Quadratic Formula and solve the equation.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$r = \frac{-46.9 \pm \sqrt{(46.9)^2 - 4(-8.1)(-38.2)}}{2(-8.1)}$$
$$= \frac{-46.9 \pm \sqrt{961.93}}{-16.2}$$
$$\approx \frac{-46.9 \pm 31.01}{-16.2}$$
$$\approx 0.98 \text{ or } 4.81$$

The owner will break even if he charges \$0.98 or \$4.81 per square foot.

b. Factors of the related equation are 0.98 and 4.81.

The two numbers divide the number line into three regions $r < 0.98$, $0.98 < r < 4.81$ and $r > 4.81$. Test a value from each interval to see if it satisfies the original inequality.

Note that, the points $r = -1$ and $r = 6$ are not included in the solution. Therefore, the solution set is $\{r \mid 0.98 < r < 4.81\}$.

The owner will make a profit if the rent per square foot is between \$1 and \$5.

c. Factors of the related equation $-8.1r^2 + 46.9r - 48.2 = 0$ are 1.34 and 4.45.

The two numbers divide the number line into three regions, $r < 1.34$, $1.34 < r < 4.45$, and $r > 4.45$. Test a value from each interval to see if it satisfies the original inequality.

Note that, the points $r = -1$ and $r = 6$ are not included in the solution. Therefore, the solution set is $\{r \mid 1.34 < r < 4.45\}$.

If rent is set between \$1.34 and \$4.45 per sq ft, the profit will be greater than \$10,000.

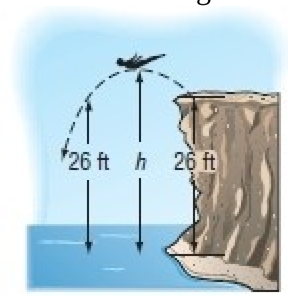
d. Factors of the related equation $-8.1r^2 + 46.9r - 48.2 = 0$ are 1.34 and 4.45.

The two numbers divide the number line into three regions $r < 1.34$, $1.34 < r < 4.45$, and $r > 4.45$. Test a value from each interval to see if it satisfies the original inequality.

Note that, the point $x = 2$ are not included in the solution. Therefore, the solution set is $r < 1.34$ and $r > 4.45$.

If rent is set between \$0 and \$1.34 or above \$4.45 per sq ft, the profit will be less than \$10,000.

73. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet.



SOLUTION:
Substitute 26 for h in the equation and solve.

$$\begin{aligned} -16t^2 + 4t + 26 &= 26 \\ -16t^2 + 4t &= 0 \\ -4t(-4t+1) &= 0 \\ -4t + 1 &= 0 \\ t &= \frac{-1}{-4} \\ &= 0.25 \end{aligned}$$

It will take 0.25 seconds for her to return to a height of 26 feet.

Find all of the possible values of $\pm\frac{b}{a}$ for each replacement set.

74. $a = \{1, 2, 4\}$; $b = \{1, 2, 3, 6\}$

SOLUTION:
$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

75. $a = \{1, 5\}$; $b = \{1, 2, 4, 8\}$

SOLUTION:
$\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}, \pm\frac{8}{5}$

76. $a = \{1, 2, 3, 6\}$; $b = \{1, 7\}$

SOLUTION:
$\pm1, \pm7, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{7}{2}, \pm\frac{7}{3}, \pm\frac{7}{6}$